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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS (2019-20)
Course: M.Sc Mathematics (Final Year)

Important Notes: *(1)Students are advised to read the separate enclosed instructions before beginning the writing of assignments.(2)Out of 20 Internal Assignment marks per Paper, 5 marks will be awarded for the regularity (attendance) to the Counseling/ Contact Programme classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper. Answer all questions. Each question carries 05marks*

PAPER V:COMPLEX ANALYSIS

1. a) Show that there are complex numbers z satisfying $|z-a|+|z+a| \leq 2|c|$ if and only if $|a| \leq |c|$. If this condition is fulfilled, what are the smallest and largest values of $|z|$?

   b) Find the radius of convergence of the following power series.

   i) $\sum \frac{z^n}{n!}$ ii) $\sum n!\, z^n$ iii) $\sum z^{n!}$

   c) If $\sum_0^\infty a_n$ converges, then show that $f(z) = \sum_0^\infty a_n z^n$ tends to $f(1)$ as z approaches 1 in such a way that $\frac{|1-z|}{1-|z|}$ remains bounded.

2. a) Let $H(D)$ denote the set of all analytic functions defined on an open set D and $f \in H(D)$. Let z_0 be any point of D such that $f'(z_0) \neq 0$ then show that f is conformal at z_0. Further give an example for isogonal mapping which is not conformal.

   b) Show that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line.

   c) If the function $f(z)$ is analytic on a rectangle R, then show that $\int_{\partial R} f(z)dz = 0$.

3. a) State and prove the argument principle.

   b) Evaluate the following integrals.

   i) $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ ii). $\int_0^\infty \frac{dx}{(x^2+1)(x^2+9)}$

   c) Show that non-constant entire functions are unbounded.

PAPER VI: TOPOLOGY

1. a) Let X be the topological space. Suppose that $\mathcal{C}$ is a collection of open subsets of X such that for each open set U of X and each $x$in U, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Then show that $\mathcal{C}$ is a basis for the topology of X.

   b) Let $\mathbb{R}_l$ denote $\mathbb{R}$ with lower limit topology and $\mathbb{R}_K$ denote $\mathbb{R}$ with K-topology. Then show that the topologies of $\mathbb{R}_l$and$\mathbb{R}_K$are not comparable.

2. a) Let X and Y be the topological spaces. Let π_1 and π_2 be the projections of $X \times Y$ onto X and Y, respectively. Show that the collection

   $$\mathcal{S} = \{\pi_1^{-1}(U) \mid U \text{ open in } X\} \cup \{\pi_2^{-1}(V) \mid V \text{ open in } Y\}$$

   is a subbasis for the product topology on $X \times Y$.

   b) Let X be the topological space and $A, B \subset X$. Then prove the following.

   i) If $A \subset B$ then $A^0 \subset B^0$ ii) $(A \cap B)^0 = A^0 \cap B^0$ iii). $A^0 \cup B^0 \subset (A \cup B)^0$.

   Further give an example to show that equality does not hold good.

c) Prove that for functions $f: \mathbb{R} \to \mathbb{R}$, the ϵ-δ definition of continuity implies the open set definition.

3. a) Show that a path connected space X is necessarily connected. Is converse true? Justify.

   b) Show that every compact subspace of a metric space is bounded in that metric and is closed. Find a metric space in which not every closed bounded subspace is compact.

   c) Show that every well-ordered set X is normal in the order topology.

PAPER VII: MEASURE THEORY & FUNCTIONAL ANALYSIS

1. a) Prove that the interval (a, ∞) is measurable. Deduce that every Borel set is measurable.

   b) Prove that a bounded function defined on a set of finite measure is Lebesgue integrable if and only if it is measurable.

   c) Define the Lebesgue integral. Consider $(x) = \sum_{n=1}^{200} \frac{1}{n^6} \chi_{\left[0,\frac{n}{200}\right]}(x), x \in [0,1]$, where χ is the characteristic function. Find the Lebesgue integral of f on [0,1].

2. a) Let $f \geq 0$ and measurable. Show that $\exists$ a sequence $\{\varphi_n\}$ of simple functions $\ni$ $\varphi_n \uparrow f$.

   b) Define a complete metric space. Show that C[a, b] is not complete under integral metric.

   c) Prove that a metric space is compact if and only if it is sequentially compact.

3. a) If X is a finite dimensional normed linear space, then prove that any two norms on X are equivalent. Does the converse hold true? Justify.

   b) Is there any result which guarantees the uniqueness of the norm attaining point of a bounded linear operator? Discuss.

   c) Let X be a Banach space. Prove that X is reflexive if and only if X^* is reflexive.

PAPER VIII: NUMERICAL ANALYSIS

1. a) Use Bairstow's method to extract a quadratic factor of the form $x^2 - px - q$ from a polynomial Choose $p = 1, q = 2$.

   b) Explain the Householder method to reduce the following matrix in to tri-diagonal form

$$A = \begin{pmatrix} 1 & 4 & 0 \\ 4 & 1 & -4 \\ 0 & -4 & 2 \end{pmatrix}$$

2. a) Fit both linear and quadratic curve for a function $f(x) = \frac{x}{x+1}$ over an interval *[0, 1]* with respect to the weight function *w(x) = 1* using least square approximation method.

   b) Find the cubic spline interpolation polynomial in the interval [0,4] for the following data:

x	*0*	*1*	*2*	*3*	*4*
y	*3*	*3*	*9*	*27*	*63*

   Given $s''(0) = s''(4) = 0$.

3. a) Solve an initial value problem $\frac{dy}{dx} = \frac{2x}{1+y}$, $y(0) = 0$ in the range $0 \le x \le 1$ using Milne's Predictor-Corrector method, choose $h = 0.2$.

   b) Find the solution of a BVP $y'' - 2y' + 4y = \sin x$ $y(0) = 0, y(1) = 0$ using finite difference method, choose $h = 0.25$.

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