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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS (2019-20)
Course: M.Sc Mathematics (Previous)

Important Notes: *(1)Students are advised to read the separate enclosed instructions before beginning the writing of assignments.(2)Out of 20 Internal Assignment marks per Paper, 5 marks will be awarded for the regularity (attendance) to the Counseling/ Contact Programme classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper. Answer all questions. Each question carries 05marks*

PAPER I:ALGEBRA

1. a)If p is a prime number and G is a group of order p^n, $n \geq 1$ then prove that the centre of G has at least 'p' elements.

   b) Let p be a prime dividing o(G). Show that every sylow p-subgroup of G/K is of the form PK/K, where P is a sylow p-subgroup of G.

   c)Prove that the product of any two ideals of a ring R is also an ideal of R. (2+2+1)

2. a)Define an Euclidean ring. Show that the ring I of all integers is a Euclidean ring.

   b) Let F be a field. If A = {(x, y, 0): x, y ∈ F}, B = {(0, y, z): y, z ∈F} be subspaces of $F^3(F)$, find the dimension of the subspace A+B.

   c)If W is a subspace of a finite dimensional vector space V, define the annihilator A(W) of a subspace W. Further show that

   i) $A(W_1 + W_2) = A(W_1) \cap A(W_2)$

   ii) $A(W_1 \cap W_2) = A(W_1) + A(W_2)$. (1+2+2)

3. a) Let T be a linear operator on a vector space V over F. If $W_1, W_2, \ldots, W_k$ are T-invariant subspaces of V, prove that $\sum_{i=1}^{k} W_i$ and $\bigcap_{i=1}^{k} W_i$ are T-invariant subspaces of V.

b) If f(x) ∈ F[x] is irreducible over F, then show that all its roots have the same multiplicity

PAPER II:ANALYSIS-I

1. a) Prove that $|x+y|^2 + |x-y|^2 = 2|x|^2 + 2|y|^2$, if $x \in R^k$ and $y \in R^k$. Interpret this geometrically, as a statement about parallelograms.

   b) Construct a bounded set of real numbers with exactly three limits points.

   c) Prove that every connected metric space with at least two points is uncountable..

2. a) Prove that every convex subset of R^k is connected.

   b) Suppose f is differentiable on $(0,\infty)$, f'' is bounded on $(0,\infty)$ and $f(x) \to 0$, as $x \to \infty$, then prove that $f'(x) \to \infty$ as $x \to \infty$.

   c) Suppose f is bounded real function on $[a,b]$ and $f^2 \in \mathcal{R}$ on $[a,b]$. Does it follow that $f \in \mathcal{R}$? Does the answer change if we assume that $f^3 \in \mathcal{R}$?

3. a) Prove that let $\{f_n\}$ be uniformly bounded sequence of functions which are Riemannian integrable on $[a,b]$ and put $F_n = \int_a^x f_n(t)dt$, $a \leq x \leq b$ then there exists a subsequence $\{F_{n_k}\}$ Which converges uniformly on $[a,b]$.

   b) If $f(x) = 0$ for all irrational x, $f(x) = 1$ for all rational x then prove that $f \notin \mathcal{R}$ on $[a,b]$ for any $a < b$.

PAPER III:ANALYSIS-II

1. a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
   b) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of continuous functions which converges uniformly to a function f on a set E. Prove that $\lim_{n\to\infty} f_n(x_n) = f(x)$ for every sequence of points $x_n \in E$. such that $x_n \to x$ and $x \in E$. Is the converse of this true?
2. a) Consider $f(x) = \sum_{n=1}^{\infty} \frac{1}{1+n^2x}$. For what values of x does the series converges absolutely? On what intervals does it converge uniformly? On what intervals does it fail to converge uniformly? Is f continues wherever the series converges ? Is f bounded.
   b) Consider $f(x) = \sum_{n=1}^{\infty} \frac{(nx)}{n^2}$, where x is real. Find all discontinuous of f and show that they form a countable dense set. Show that f is nevertheless Riemann-integrable on every bounded interval.
   c) Let $x_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n}$, and define $y_n = x_n - \log n$. Show that the sequence (y_n) tends to a limit y. Where $0 < y \leq 1$. Deduce that $1 - \frac{1}{2} + \frac{1}{3} - \cdots = \log 2$.
3. a) If the partial derivatives f_x and f_y exists and are bounded in a region $R \subset R^2$, then f is continuous in R.
   b) If $f(0,0) = 0$ and $f(x,y) = \frac{xy}{x^2+y^2}$ if $(x,y) \neq (0,0)$. Prove that $(D_1 f)(x,y)$ and $(D_2 f)(x,y)$ exists at every point of R^2, although f is not continuous at $(0,0)$
   c) Take $m = n = 1$ in the implicit function theorem and interpret the theorem graphically.

PAPER IV:DIFFERENTIAL EQUATIONS

1. a) Find the transformation which transforms $a_0(t)x'' + a_1(t)x' + a_2(t)x = 0$ into an equation whose in the first derivative term is absent.
   b) Show that the function $\{t^3, |t^3|\}$ are linearly independent on $[-1,1]$ but not on $[-1,0]$
2. a) Given a solution of $(1-t^2)x'' - 2tx' + 6x = 0, \phi_1(t) = 3t^2 - 1$. Find its general solution.
   b) Solve $x^{(4)} + 4x = 2\sin t + 4e^t + 1 + 3t^2$ by using the method of undetermined co-efficients.
3. a) Solve the nonlinear equation $p^2 - 3q^2 - u = 0$ with Cauchy data $u(x,0) = x^2$ using Cauchy method of characteristics.
   b) Find the solution of the heat equation of $u_t = c^2 u_{xx}$; $0 < x < l$; $0 < t < \alpha$ when subjected to the Neumann conditions $u(0,t) = k_1$, $u(l,t) = k_2$; and an initial condition $u(x,0) = \phi(x)$ for all x.

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