

**Q.P. Code – 50722**

**Second Year B.Sc. Degree Examination**

**OCTOBER/NOVEMBER 2014**

**(Directorate of Distance Education)**

**(DSB 230) Paper II – MATHEMATICS**

*Time : 3 Hours]*

*[Max. Marks : 90*

**Instructions to Candidates :**

*Answer any **SIX** full questions of the following choosing atleast **ONE** from each Part.*

PART – A

1. (a) (i) Find the order and degree of differential equation  
$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^4 = e^{4x}. \quad \mathbf{2}$$
- (ii) Solve  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0.$  **2**
- (b) Solve  $\frac{dy}{dx} = \frac{6x - 2y - 7}{3x - y + 4}.$  **5**
- (c) Solve  $y(2x - y + 1)dx + x(3x - 4y + 3)dy = 0.$  **6**
2. (a) (i) Solve  $P^2 - 3P + 2 = 0.$  **2**
- (ii) Find the general and singular solution of  $y = xP + P^2.$  **2**
- (b) Solve  $16x^2 + 2P^2y - P^3x = 0.$  **5**
- (c) Find orthogonal trajectories of the family  $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$ , where 'λ' is being a parameter. **6**

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### PART – B

3. (a) (i) Solve  $(D^2 + 9)y = \cos 3x$ , where  $D = \frac{d}{dx}$ . **2**
- (ii) Solve  $(4D^3 + 4D^2 + D)y = 0$ , where  $D = \frac{d}{dx}$ . **2**
- (b) Solve  $(D^4 - 18D^2 + 81)y = 36e^{3x}$ , where  $D = \frac{d}{dx}$ . **5**
- (c) Solve  $(D^2 - 4D + 3)y = 2xe^{3x}$ , where  $D = \frac{d}{dx}$ . **6**
4. (a) (i) Evaluate  $\lim_{x \rightarrow 0} \frac{x \cdot \sin x}{x^3}$ . **2**
- (ii) Evaluate  $\lim_{x \rightarrow 0} (\operatorname{cosec} x - \cot x)$ . **2**
- (b) State and prove Cauchy's mean value theorem. **5**
- (c) Expand  $\log(\sec x)$  upto the term containing  $x^6$  using Maclaurin's series. **6**

### PART – C

5. (a) (i) Show that in a group  $G$ ,  $(a^{-1})^{-1} = a$ . **2**
- (ii) Find the distinct generators of the cyclic group of order 15. **2**
- (b) Prove that every subgroup of a cyclic group is cyclic. **5**
- (c) State and prove Fermat's theorem. **6**
6. (a) (i) Solve  $x - 6 < 2x - 5 \leq x - 3$ . **2**
- (ii) For any two real numbers  $x$  and  $y$  show that  $|x - y| \geq |x| - |y|$ . **2**
- (b) Find the order of the permutation and find whether they are odd or even.  
 $\phi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 4 & 5 & 1 & 7 & 2 & 8 & 3 & 9 & 6 \end{pmatrix}$ . **5**
- (c) Find the envelopes of family of lines  $x \cdot \cos^3 \alpha + y \cdot \sin^3 \alpha = a$ , where ' $\alpha$ ' is a parameter. **6**

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PART – D

7. (a) (i) Find whether the sequence  $\left\{\frac{n}{n+1}\right\}$  is bounded or not. **2**

(ii) Examine the nature of the sequence  $\left\{\left(\frac{n}{n+1}\right)^n\right\}$ . **2**

(b) If the sequence  $\{x_n\}$  converges to  $l$  and  $\{y_n\}$  converges to  $m$  then show that  $\left\{\frac{x_n}{y_n}\right\}$  converges to  $\frac{l}{m}$ . **5**

(c) Prove that every convergent sequence is bounded. **6**

8. (a) (i) Show that  $\sum \frac{1}{(\log n)^n}$  series is convergent. **2**

(ii) Verify whether series  $1^2 + 2^2 + 3^2 + 4^2 + \dots$  converges or not. **2**

(b) State and prove D'Alembert's Ratio test. **5**

(c) Find sum to infinity of the series

$1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \dots$  **6**

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